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Technical Notes

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Internal Stresses in Composite Laminates Due to Cyclical Hygrothermal Loading

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I. Introduction

THE application of composite materials to the supersonic technology is not as straightforward as it could seem at first sight, mainly because of the specificity of the environmental conditions related to a supersonic flight.

During a supersonic flight, external structures are subjected to temperature and moisture variations, which are cyclical to a good approximation. During the subsonic part of the flight, temperatures are quite low, whereas in the supersonic regime the aerodynamic friction introduces consistent heating, raising the temperatures. External humidity conditions vary from humid, on the ground, to dry as soon as the altitude raises.

Such variable environmental conditions produce internal hygrothermal distributions and stress, which are far from being uniform, as a result of the diffusion processes that are relatively slow compared to a time of a flight. Other accompanying mechanisms, which can be very complex, are matrix degradation, diffusion-stress-damage interaction, stress relaxation, and matrix cracking.[1,2] However, calculation of moisture concentration and internal stress due to complex environmental conditions is the very first step for approaching rationally all of these phenomena.

Calculation of internal stresses due to transient hygrothermal fields has been performed for plate and cylindrical structures[3–5]; a closed-form solution to the problem of a pipe subjected to cyclical conditions has been also given.[6]

In the present Note the moisture concentration fields and internal stresses induced by cyclical hygrothermal conditions as a result of supersonic flights are calculated by employing the Fick's law and the classical laminated plate theory, adapted for taking into account transient conditions. Simulations contribute to the identification of critical structural zones and service conditions and constitute a useful tool for the simulation of accelerated cycles, in view of experimental applications.

II. Model Description

Moisture Concentration Field

Jacquemin and Vautrin gave a closed-form solution[7] for the concentration fields in cylindrical structures subjected to cyclical conditions within the framework of the Fick's theory. We should recall that external fluctuations of temperature and moisture influence the external boundary conditions in terms of concentration but also the moisture diffusion coefficient D, which is temperature dependent. The thermal equilibrium is supposed instantaneous, so that the thermal field inside the plate is uniform.

The concentration field exhibit two solutions: a transient then permanent solution in the internal regions of the pipe and a fluctuating then periodic solution in regions close to the lateral surfaces. The extent e_0 of the fluctuating then periodic solution is given by (see also Refs. 8 and 9)

$$e_0 = 2\sqrt{\pi \int_0^\tau D(t)\,\mathrm{d}t} \tag{1}$$

where τ is the period of a cycle.

The direct analytical method proposed in Ref. 7 for the concentration fields is able to describe accurately only the transient solution, whereas for regions close to the surfaces a finite difference scheme is employed.

In the present Note a similar approach is proposed for plate structures. A laminated plate of infinite in-plane dimensions and thickness e is considered. The analytical method relies on the hypothesis of unidimensional Fickian diffusion, where the moisture diffusion coefficient depends on the temperature only through the Arrhenius law

$$D(t) = A\exp[B/T(t)] \tag{2}$$

where D is the diffusion coefficient, T is the temperature, t is the time, and A and B are constants.

The moisture concentration field is the solution of the equation

$$c(z,t)_{,t} = D(t)c(z,t)_{,zz} \qquad \forall 0 < z < e, \qquad t > 0 \tag{3}$$

$$\begin{aligned} c(0,t) = c(e,t) &= c_\infty(t) \\ c(z,0) &= c_i(z) \end{aligned} \tag{4}$$

where z is the coordinate in the thickness direction, $c_i(z)$ is the initial concentration, and $c_\infty(t)$ is the concentration at the lateral surfaces, depending on the external relative humidity $HR(t)$ by the law

$$c_\infty(t) = C \cdot HR(t)^b \tag{5}$$

C and b are constants. All of the preceding functions have the same period τ. By introducing the following change of variable:

$$u(t) = \int_0^t D(q)\,\mathrm{d}q \cdot \left[\int_0^\tau D(q)\,\mathrm{d}q\right]^{-1} = \frac{\mathrm{D}(t)}{\Delta(\tau)} \Rightarrow \mathrm{d}u = \frac{\mathrm{D}(t)}{\Delta(\tau)}\,\mathrm{d}t \tag{6}$$

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problem (3) becomes

$$c(z,u)_{,u} = \Delta(\tau)c(z,u)_{,zz} \qquad \forall 0<z<e, \qquad u>0 \tag{7}$$

$$c(0,u) = c(e,u) = c_\infty(u), \qquad c(z,0) = c_i(z) \tag{8}$$

where $c_\infty(u)$ is cyclical of period 1. In problem (7) boundary conditions (8) are still time dependent; thus, the mobile average concentration has to be introduced:

$$\bar{c}(z,u) = \int_{u-1}^{u} c(z,q)\,\mathrm{d}q \tag{9}$$

The mobile average concentration is solution of the following problem:

$$\bar{c}(z,u)_{,u} = \Delta(\tau)\bar{c}(z,u)_{,zz} \qquad \forall 0<z<e, \qquad u>0 \tag{10}$$

$$\bar{c}(0,u) = \bar{c}(e,u) = \bar{c}_\infty(u), \qquad \bar{c}(z,0) = \bar{c}_i(z) \tag{11}$$

Now boundary conditions (11) are time independent, and Eq. (10) can be easily solved by employing the technique of separations of variables or Laplace transform.[9] The solution of Eq. (10) with boundary conditions (11) can be expressed as a function of cycles by employing a recursive technique. Details can be found in Ref. 7.

In what follows N will be indicated as the number of cycles at which the internal regions of the plate reach a permanent state. The concentration field inside a plate subjected to cyclical hygrothermal conditions after $(N-k)$ cycles is finally given by

$$\begin{aligned} c(z,N-k) = \hat{c}_\infty + \frac{\Delta(\tau)\pi^2}{2(e/2)^3}\sum_{n=0}^{\infty}\sum_{i=0}^{k-1}\Bigg\{(2n+1)^2 \\ \times \exp\left[-\frac{(2n+1)^2\pi^2\Delta(\tau)(N-i)}{4(e/2)^2}\right]\Bigg\}\cos\left[\frac{(2n+1)\pi z}{e}\right] \\ \times\left\{\frac{2(e/2)(-1)^{n+1}\hat{c}_\infty}{(2n+1)\pi} + \int_0^{e/2} c_i(z')\cos\left[\frac{(2n+1)\pi z'}{2(e/2)}\right]\mathrm{d}z'\right\} \\ \forall -\frac{e}{2} \le z \le \frac{e}{2} \end{aligned} \tag{12}$$

where

$$\Delta(\tau) = \int_0^\tau D(t)\,\mathrm{d}t, \qquad \hat{c}_\infty = \frac{1}{\Delta(\tau)}\int_0^\tau D(t)c_\infty(t)\,\mathrm{d}t \tag{13}$$

When, $c_i(z) = 0$, Eq. (12) becomes

$$\begin{aligned} c(z,N-k) = \hat{c}_\infty - \frac{4\hat{c}_\infty\pi}{\beta^2}\sum_{n=0}^{\infty}\sum_{i=0}^{k-1}\Bigg\{(-1)^n(2n+1) \\ \times \exp\left[-\frac{(2n+1)^2\pi^2(N-i)}{\beta^2}\right]\Bigg\}\cos\left[\frac{(2n+1)\pi z}{e}\right] \\ \forall -\frac{e}{2} \le z \le \frac{e}{2} \end{aligned} \tag{14}$$

where

$$\beta = \sqrt{e^2/\Delta(\tau)} \tag{15}$$

Series (12) and (14) converge very slowly for a small number of cycles. It must be noticed that Eqs. (12) and (14) describe correctly the solution in the interior regions of the plate only because they employ average concentration values at the boundaries [Eq. (13)]. To better represent the concentration distribution in regions close to the lateral surfaces, the finite difference method is employed besides the analytical one.

Stress Field

Stresses are calculated by using the classical lamination plate theory (see Ref. 10 for instance), adapted for taking into account transient moisture concentration fields, such as those given by Eqs. (12) and (14).

In a global Cartesian reference frame (x, y, z), strains are given by the following equations:

$$\boldsymbol{\varepsilon} = \begin{bmatrix}\varepsilon_{xx}(z)\\ \varepsilon_{yy}(z)\\ \gamma_{xy}(z)\end{bmatrix} = \begin{pmatrix}\varepsilon^0_{xx} - zw_{,xx}\\ \varepsilon^0_{yy} - zw_{,yy}\\ \gamma^0_{xy} - 2zw_{,xy}\end{pmatrix} = \begin{pmatrix}\varepsilon^0_{xx} + k_{xx}z\\ \varepsilon^0_{yy} + k_{yy}z\\ \gamma^0_{xy} + k_{xy}z\end{pmatrix} \tag{16}$$

where w is the out-of-plane displacement, k_{xx} and k_{yy} are the curvatures along the x and y directions respectively, and k_{xy} is the twist curvature. Equation (16) is fully satisfactory for describing the strain field in a symmetric laminate subjected to symmetric hygrothermal distributions.

The hypothesis of plane stress translates into the following stress–strain relationship:

$$\boldsymbol{\sigma} = \begin{bmatrix}\sigma_{xx}(z)\\ \sigma_{yy}(z)\\ \sigma_{xy}(z)\end{bmatrix} = Q\cdot(\varepsilon - \varepsilon_{\mathrm{HT}}) \tag{17}$$

where the matrix associated to the stiffness tensor Q, written in the global reference frame (x, y, z), is given by

$$[Q] = \begin{bmatrix} Q_{xxxx} & Q_{xxyy} & Q_{xxxy}\\ Q_{xxyy} & Q_{yyyy} & Q_{yxxx}\\ Q_{xxxy} & Q_{yxxx} & Q_{xyxy}\end{bmatrix} \tag{18}$$

$$\varepsilon_{\mathrm{HT}}(z) = \begin{pmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{pmatrix}\Delta T + \begin{pmatrix}\beta_x\\ \beta_y\\ \beta_{xy}\end{pmatrix} m(z) \tag{19}$$

are the hygrothermal free strains, in which $(\alpha_x\ \alpha_y\ \alpha_{xy})^T$ and $(\beta_x\ \beta_y\ \beta_{xy})^T$ are the coefficients of thermal and hygroscopic expansion of a ply, ΔT is the temperature differential from a reference state, the stress-free state, and $m(z) = m_{\mathrm{water}}/m_{\mathrm{dry}} = c(z)/\rho_s$, ρ_s being the density of the dry material.

In a ply local reference frame (1, 2, 3), the stiffness tensor Q reads

$$[Q] = \begin{bmatrix} Q_{1111} & Q_{1122} & 0\\ Q_{1122} & Q_{2222} & 0\\ 0 & 0 & Q_{1212}\end{bmatrix} = \begin{bmatrix} \dfrac{E_1}{1-\nu_{12}\nu_{21}} & \dfrac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0\\ \dfrac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & \dfrac{E_2}{1-\nu_{12}\nu_{21}} & 0\\ 0 & 0 & G_{12}\end{bmatrix} \tag{20}$$

where $\nu_{21} = \nu_{12}(E_2/E_1)$.

When $m(z)$ is given by Eqs. (12) or (14), the hygrothermal-free strains and the total strains in Eq. (17) do not satisfy compatibility equations. To overcome this difficulty, the concentration field (12) or (14) or those coming from a finite difference scheme are approximated by linear fields of the type

$$c(z) = a_i z + b_i \tag{21}$$

over spatial regions of thickness e_i smaller than a ply, called subplies and with

$$a_i = (c_i - c_{i-1})/e_i, \qquad b_i = c_{i-1} - [(c_i - c_{i-1})/e_i]z_{i-1} \tag{22}$$

The resultant force and moment caused by the stress fields (17) are given by

$$\begin{aligned} \boldsymbol{N} &= \int_0^e \boldsymbol{\sigma}(z)\,\mathrm{d}z = A\boldsymbol{\varepsilon}_0 + B\boldsymbol{k} - \boldsymbol{N}_{\mathrm{HT}} \\ \boldsymbol{M} &= \int_0^e \boldsymbol{\sigma}(z)z\,\mathrm{d}z = B\boldsymbol{\varepsilon}_0 + D\boldsymbol{k} - \boldsymbol{M}_{\mathrm{HT}} \end{aligned} \tag{23}$$

where

$$A = \int_0^e Q\,\mathrm{d}z, \qquad B = \int_0^e Qz\,\mathrm{d}z, \qquad D = \int_0^e Qz^2\,\mathrm{d}z \quad (24)$$

$$\boldsymbol{N}^{\mathrm{HT}} = \int_0^e Q\varepsilon_{\mathrm{HT}}\,\mathrm{d}z, \qquad \boldsymbol{M}^{\mathrm{HT}} = \int_0^e Q\varepsilon_{\mathrm{HT}} z\,\mathrm{d}z \qquad (25)$$

It has to be emphasized that resultant forces and moments of hygroscopic nature (25) reduce to summations over the total number of subplies.

Midplane strains and curvatures can be found by inverting relations (23); stresses are then given by Eq. (17). The employment of subplies presents some clear shortcomings. As the number of subplies is finite, the concentration and the stress fields are approximate: the accuracy increases as the number of subplies increases, but the maximum number of subplies itself is limited by some physical consideration. In the present Note a macroscopic approach is proposed, and, physically, a subply must be able to represent the homogenized behavior of the material, this imposes a limitation on its minimum dimensions.

III. Application to Supersonic Flight Cycles

A hygrothermal cycle representative of supersonic flight is presented in Fig. 1. It consists of a part at room temperature and ground humidity conditions (point A), a descent to −55°C and 0% relative humidity (HR) simulating the subsonic phase of the flight (point B), and a temperature dwell at 130°C 0% HR simulating the supersonic phase of the flight (point C). Point D is located at the end of a flight.

Simulations include an initial conditioning at 23°C and 50% HR for about three months reproducing a long-term ground maintenance, which leads to a pseudohumid state.

Simulations are performed in a 4 mm $[0_4/90_4]_s$ composite plate, each ply block being 1 mm thick. Moisture concentration fields are calculated by the finite difference method; by dividing the spatial domain in subdomains 0.1 mm thick, such subdivision is sufficient to give a good accuracy of the calculated concentration fields. The analytical solution for cycle 50 is also provided. The stress profiles are calculated by using the moisture concentration field arising from the finite difference solution in order to represent correctly the fluctuations close to the lateral boundaries.

The dry mechanical state is not free of stress; residual stresses due to manufacturing are present in the structure. The stress-free state is supposed to be achieved at a temperature of 210°C, which is around the final T_g of a high-temperature composite system, the temperature differential is then given by $\Delta T = -187$°C.

Figures 2 and 3 show the moisture concentration fields inside the plate at points A, B, C, and D of cycles 1 and 50, respectively. Hygroscopic and mechanical properties are given in Tables 1 and 2, respectively; they are representative of a IM7/977-2 carbon fiber-epoxy composite system.

The average effect of the supersonic cycles consists of drying the plate until a quasi-dry state, which is reached after around 300 cycles. Fluctuating boundary conditions are felt within zones that are close to the external surfaces and around 0.5 mm thick, while internal regions of the plate are in a transient regime. When the quasi-dry state is attained, internal regions reach a permanent regime (uniform profile), while the zones within e_0 keep fluctuating in a periodic fashion. The analytical solution, as already cited in Sec. II, respects average boundary conditions and is adequate for the internal regions only.

As e_0 depends on the cycle and the material only, it is clear that a plate thinner than $2e_0$ is never in a permanent state, fluctuations extend over the whole thickness. Figures 4 and 5 illustrate stresses σ_{xx} and σ_{yy} at points A, B, C, and D of cycle 1. Figures 6 and 7 show the evolution of internal stresses with increasing number of cycles.

From the figures, several things can be noted:

1) The level of internal stress is already high after manufacturing. Residual curing stresses are reported for reference in each figure and are pictured in continuous lines. These stresses are uniform through the thickness of the plate, by the assumption of uniform ΔT, and

Table 1 Hygroscopic properties

Property	Value
A [Eq. (2)], mm^2/h	7.18
B [Eq. (2)], K^{-1}	−2910.2
C [Eq. (5)], %	0.0007
b [Eq. (5)]	1.6036

Table 2 Material properties used in the simulations (IM7/977-2)

Property	Value
E_1, GPa	152
E_2, GPa	8.4
G_{12}, GPa	4.2
ν_{12}	0.35
α_1 (ε^0C^{-1})	0.09×10^{-6}
α_2 (ε^0C^{-1})	28.8×10^{-6}
β_1	0
β_2	0.6

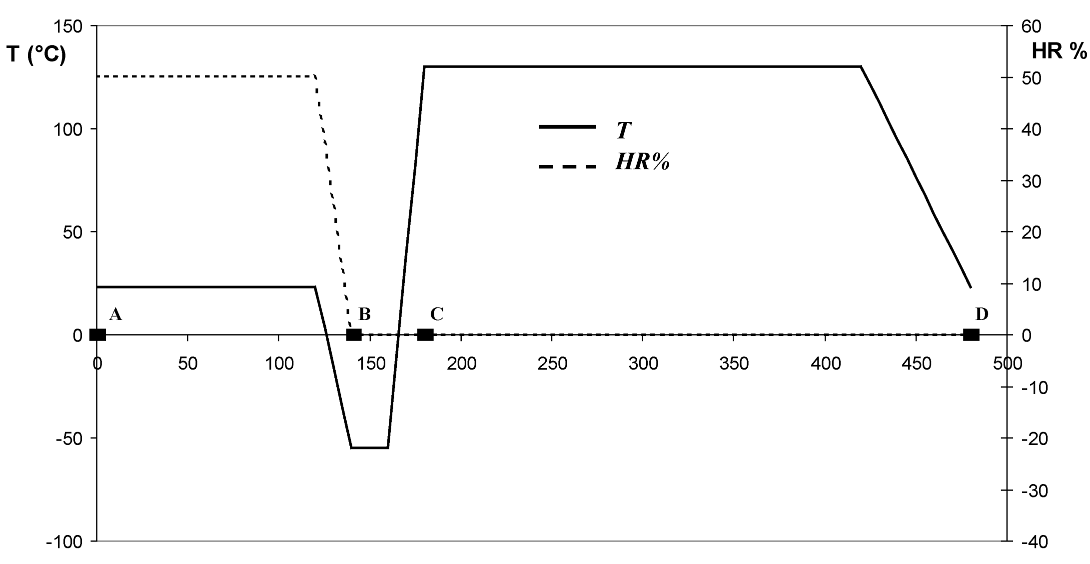

Fig. 1 Hygrothermal cycle.

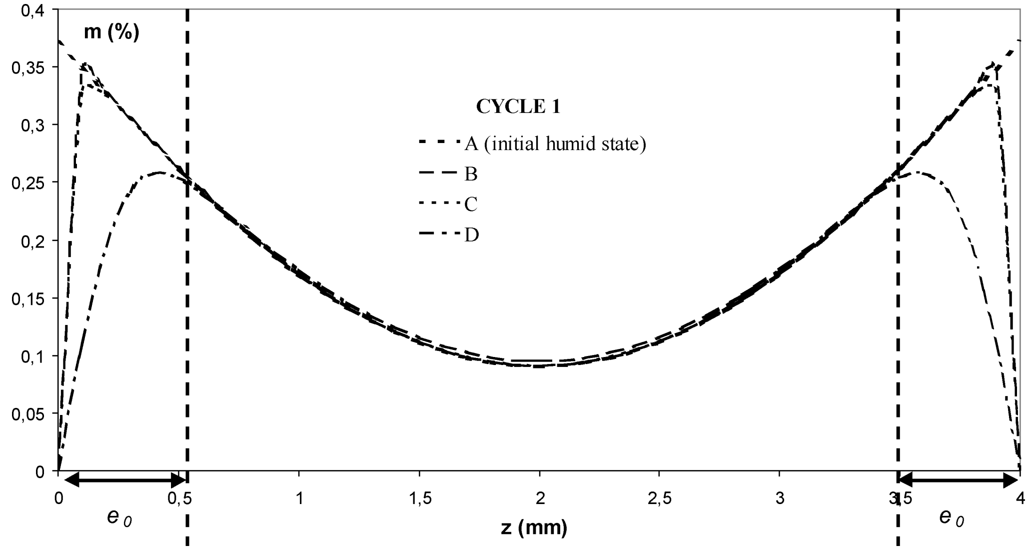

Fig. 2 Moisture concentration during cycle 1.

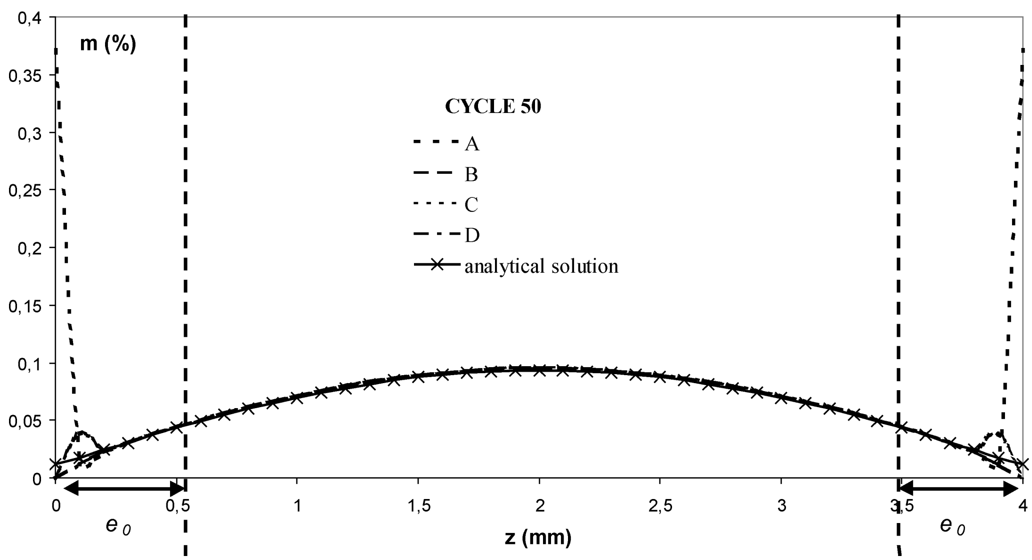

Fig. 3 Moisture concentration during cycle 50.

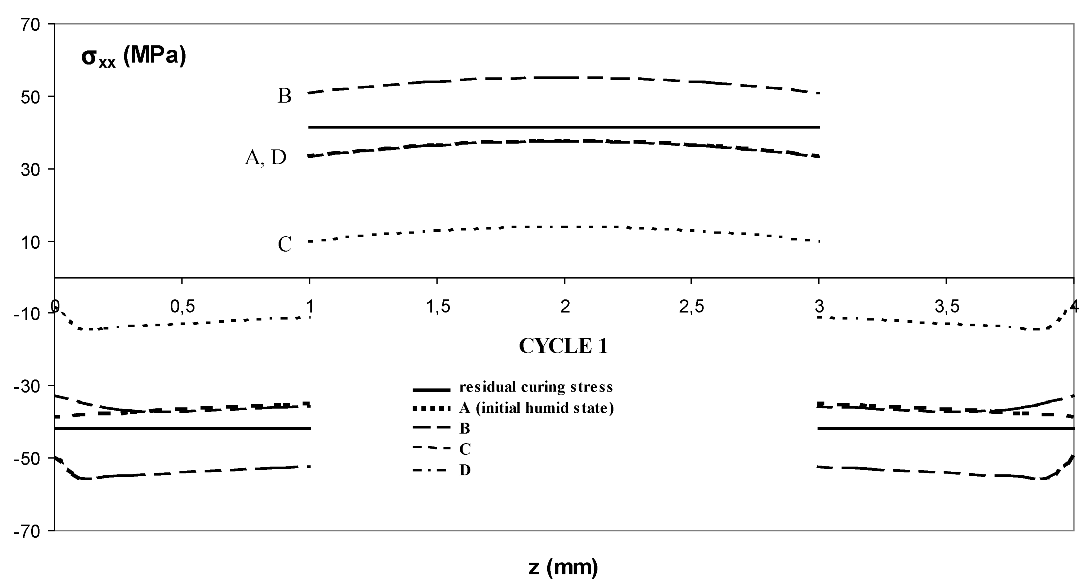

Fig. 4 Internal stresses σ_{xx} during cycle 1.

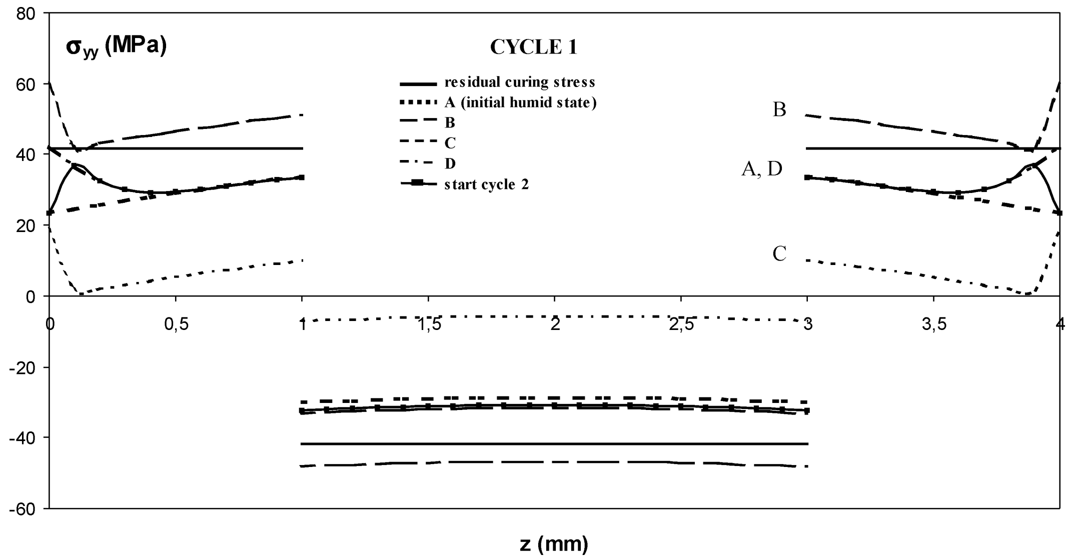

Fig. 5 Internal stresses σ_{yy} during cycle 1.

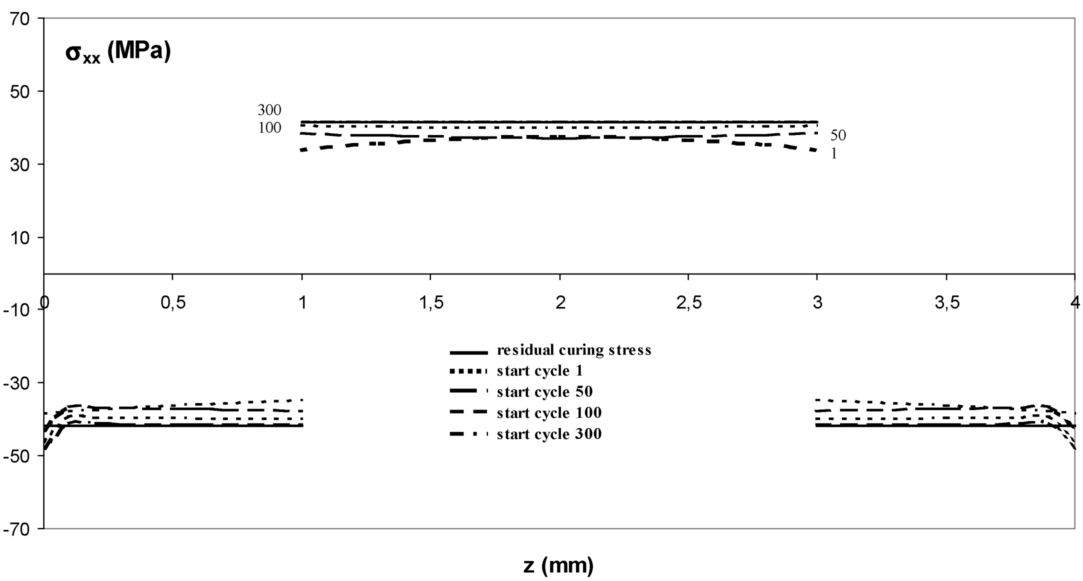

Fig. 6 Internal stresses σ_{xx} at the start of cycles 1, 2, 50, 100, and 300.

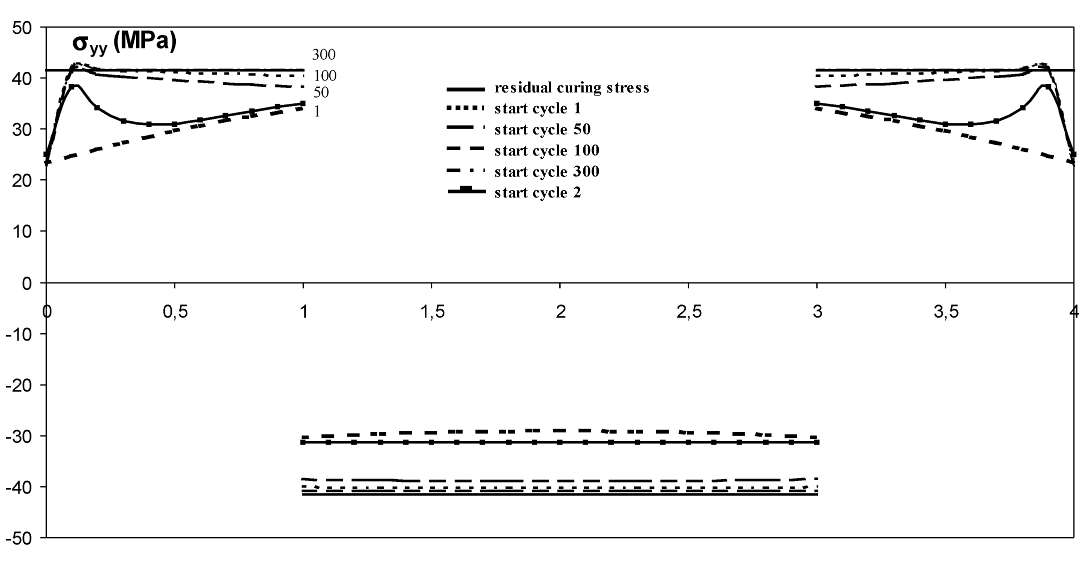

Fig. 7 Internal stresses σ_{yy} at the start of cycles 1, 2, 50, 100, and 300.

take a value of around 42 MPa. It should be emphasized that σ_{xx} and σ_{yy} stresses are tensile, respectively, in the 90- and 0-deg plies, thus, in both cases, in directions transverse to the fibers. Usually the direction transverse to the fibers is also the direction with the poorest material strength.

2) The internal stresses that arise after a long period of maintenance caused by the absorption of moisture, point A in cycle 1, counteract only partially the residual curing stress. The average tensile value of internal stress is around 35 MPa, which is relatively high when compared to the dry reference state established after cure. This result, which might seem rather peculiar to the case under study, can be generalized: in fact, polymeric resins for aeronautical applications tend to absorb a very few percent of water such that even a long period of maintenance does not have much effect on the initial residual state of stress.

3) Thermal fluctuations from $-55°C$ to $130°C$, points B and C, respectively, extend over the whole thickness of the plate and exhibit an average amplitude of around 40 MPa; the mean stress is about 30 MPa. Low-temperature conditions are quite critical; stresses can reach, over the cycling, local values as high as 60 MPa.

4) Hygroscopic fluctuations induce stress gradients over a region close to the lateral surfaces and of thickness e_0 and fluctuating stresses, whose magnitude can be as high as 40 MPa. The fluctuating regime established between point B of a cycle and the start of the subsequent cycle (see Fig. 5 for instance, relative to cycle 1) is peculiar to hygroscopic fluctuations.

5) Drying induced by supersonic cycles promotes a progressive resurgence of the residual curing stresses; the mean of all fluctuations increases with increasing cycles.

IV. Conclusions

The present Note presents simulations of moisture concentration and stress fields promoted by cyclical hygrothermal conditions as a result of supersonic flight in carbon-epoxy composite plates.

Calculations are based on very straight hypotheses, the moisture concentration fields are obtained by employing the unidimensional Fick's diffusion law, and internal stresses are calculated by adapting the classical lamination plate theory to transient conditions.

Concentration and stress fields are in a transient then permanent regime over interior regions of the plate, whereas fluctuating then periodic conditions prevail over a zone close to the lateral surfaces and of thickness e_0.

The value of e_0 depends on material properties and cycle characteristics and is constant with the number of cycles. For the case under study e_0 is around 0.5 mm.

As the supersonic cycles promote drying from initial humid conditions, stresses evolve toward an "after-manufacturing" condition, with progressive resurgence of residual curing stresses. The mean of all fluctuations increases with increasing cycles: in particular, zones close to the external surfaces are exposed to aggressive mixed thermal-hygroscopic stress fluctuations of amplitude as high as 40 MPa.

The present Note represents a first effort toward a rational understanding of degradation and damage caused by hygrothermal cycling. Future developments include enriching the model with damage laws and, most of all, providing experimental evidence of hygrothermal fatigue. Interaction between the enriched model and experimental results is foreseen and will be the object of future publications.

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